Polyakov Loop Models, Z(N) Symmetry, and Sine-Law Scaling

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We construct an effective action for Polyakov loops using the eigenvalues of the Polyakov loops as the fundamental variables. We assume $Z(N)$ symmetry in the confined phase, a finite difference in energy densities between the confined and deconfined phases as $T \rightarrow 0$, and a smooth connection to perturbation theory for large T. The low-temperature phase consists of $N - 1$ independent fields fluctuating around an explicitly $Z(N)$ symmetric background. In the low-temperature phase, the effective action yields non-zero string tensions for all representations with non-trivial N-ality. Mixing occurs naturally between representations of the same N-ality. Sine-law scaling emerges as a special case, associated with nearest-neighbor interactions between Polyakov loop eigenvalues.

Effective actions for the Polyakov loop are directly relevant to the phase diagram and equation of state for QCD and related theories [1, 2, 3, 4, 5]. There is also good reason to believe that Polyakov loop effects play an important role in chiral symmetry restoration [6, 7, 8, 9, 10]. Furthermore, there may be a close relationship between the correct effective action and the underlying mechanisms of confinement [11, 12, 13, 14, 15].

In this letter, we construct a general effective action for the Polyakov loop, making as few assumptions as possible. Our goal is a comprehensive model which describes both the confined and deconfined phases of $SU(N)$ pure gauge theories at finite temperature, as well as the properties of the deconfining phase transition. We will describe below the general form of such an effective action, using the Polyakov loop eigenvalues as natural variables. Effective actions of this type have previously been constructed for QCD at high temperature [16, 17, 18], QCD in two dimensions[19, 20], and for lattice gauge theories at strong coupling[21, 22, 23, 24].

The Polyakov loop $P(x)$ is the natural order parameter for the deconfinement transition. It is defined as a path-ordered exponential $P(x) = P \exp \left[i \int_0^\beta d\tau A_0 (\tau, x) \right]$ where x is a d-dimensional spatial vector and β is the inverse temperature. The confined phase has a $Z(N)$

symmety that implies $\left\langle Tr_F\ P^k(x)\right\rangle = 0$ for all n not divisible by N: $k|N \neq 0$. For these powers of the Wilson line, the asymptotic behavior

$$\left\langle Tr_F P^k(x)\, Tr_F P^{+k}(y)\right\rangle \propto \exp\left[-\beta\sigma_k\,|x-y|\right] \tag{1}$$

is observed in lattice simulatiosn as $|x-y| \to \infty$. The string tension σ_k is in general temperature-dependent. Similar behavior is also seen for the two point functions associated with irreducible representations of the gauge group $\langle Tr_R P(x)\, Tr_R P^+(y)\rangle$. Every irreducible representation R has an associated N-ality k_R such that $Tr_R P \to z^{k_R} Tr_R P$ under a global $Z(N)$ transformation, with $k_R \in \{0,..,N-1\}$ and $z \in Z(N)$. Any representation with $k_R \neq 0$ gives an operator $Tr_R P$ which is an order parameter for the spontaneous breaking of the global $Z(N)$ symmetry.

It is widely held that the asymptotic string tension depends only on the N-ality of the representation. In fact, lattice data from simulations of four dimension $SU(3)$ gauge theory do not yet show this behavior [25, 26]. Instead, the string tension σ_R associated with a representation R scales approximately as

$$\sigma_R = \frac{C_R}{C_F}\sigma_F \tag{2}$$

where C_R is the quadratic Casimir invariant for the representation R. This behavior can be rigourously demonstrated in two-dimensional gauge theories. For simplicity, we will refer to this as Casimir scaling. We use the term $Z(N)$ scaling to descibe the asymptotic behavior

$$\sigma_R = \frac{k\,(N-k)}{N-1}\sigma_F \tag{3}$$

where k is the N-ality of the representation R. As we will show below, $Z(N)$ scaling is obtained from Casimir scaling at large distances if there is mixing between representations of the same N-ality. If $Z(N)$ scaling holds, there are $[N/2]$ distinct asymptotic string tensions, where $[N/2]$ is the largest integer less than or equal to $N/2$. For the first $[N/2]$ antisymmetric representations made by stacking boxes in Young tableux, Casimir and $Z(N)$ scaling are identical. Another possible scaling law for the string tensions is sine-law scaling

$$\sigma_R = \frac{\sin\left(\frac{\pi k}{N}\right)}{\sin\left(\frac{\pi}{N}\right)}\sigma_F \tag{4}$$

which has been shown to occur in softly broken $N = 2$ super Yang-Mills theories[27] and in MQCD[28].

In a gauge in which A_0 is time independent and diagonal, we may write P in the fundamental representation as

$$P_{jk} = \exp\left(i\theta_j\right)\delta_{jk} \tag{5}$$

where we shall refer to the N numbers θ_j as the eigenvalues. They are not independent because $\det\left(P\right) = 1$ implies

$$\sum_j \theta_j = 0 \, mod \, 2\pi. \tag{6}$$

The information in the different representations is redundant. All the information is contained in the $N - 1$ independent eigenvalues of P.

Without spatial gauge fields, the only gauge-invariant operators we can construct are class functions of P, depending solely on its eigenvalues. Thus the effective action should take P to be in the Cartan, or maximally commuting, subgroup $U(1)^{N-1}$. An effective action constructed from $SU(N)$ matrices would introduce spurious Goldstone bosons associated with the off-diagonal components of P in the deconfined phase. On the other hand, the simplest Landau-Ginsburg models of the $SU(2)$ and $SU(3)$ deconfining transitions have used $Tr_F P$ as the basic field. These two cases are special because $Tr_F P$ specifies the Polyakov loops in all other representations. In $SU(4)$, there are sets of eigenvalues for which

$$Tr_F P = 0 \quad \text{and} \quad \text{Tr}_\text{F}\text{P}^2 = 0 \tag{7}$$

consistent with a $Z(4)$ symmetry, and another set for which

$$Tr_F P = 0 \quad \text{and} \quad \text{Tr}_\text{F}\text{P}^2 \neq 0 \tag{8}$$

consistent with a $Z(2)$ symmetry. Thus knowledge of $Tr_F P$ alone is insufficient for $N > 3$[3]. From the characteristic polynomial, one may show that the eigenvalues of a special unitary matrix are determined by the set $\left\{Tr_F P^k\right\}$ with $k = 1..N - 1$, and of course *vice versa*.

In both the confined and deconfined phases, we would like our effective theory to proceed from a classical field configuration which has the symmetries of the phase. If we denote that field cofiguration as P_0 in the confined phase, $Z(N)$ symmetry requires that $Tr_F P_0^k = 0$ for all k not divisible by N. Enforcing this requirement for $k = 1$ to $N - 1$ leads to a unique set of eigenvalues via the characteristic equation $z^N + (-1)^N = 0$, but it is instructive to derive

the set another way. For temperatures T below the deconfinement transition T_d, center symmetry is unbroken. Unbroken center symmetry implies that zP_0 is equivalent to P_0 after an $SU(N)$ transformation:

$$zP_0 = gP_0g^+. \tag{9}$$

This condition in turn implies $Tr_R P_0 = 0$ for all representations R with non-zero N-ality, which means that all representations with non-zero N-ality are confined. The most general form for P_0 may be given as hdh^+, where $h \in SU(N)$, and d is the diagonal element of $SU(N)$ of the form

$$d = w \; diag\left[z, z^2, .., z^N = 1\right] \tag{10}$$

where z is henceforth $\exp\left(2\pi i/N\right)$, the generator of $Z(N)$. The phase w ensures that d has determinant 1, and is given by $w = \exp\left[-(N+1)\pi i/N\right]$. Strictly speaking, w is required only for N even, but it is convenient to use it consistently. We will henceforth identify P_0 with d. Another useful representation is $(P_0)_{jk} = \delta_{jk} \exp\left[i\theta_j^0\right]$ where

$$\theta_j^0 == \frac{\pi}{N}\left(2j - N - 1\right). \tag{11}$$

Thus the $Z(N)$-symmetric arrangement of eigenvalues is uniform spacing around the unit circle. This is consistent with the known large-N behavior of soluble models in the confined phase[29].

Assume that P_0 is the global minimum of the potential V associated with the effective action for temperatures less than the deconfining temperature T_d. Because the gauge fields transform as the adjoint representation, V is a class function depending only on representations of zero N-ality. It is thus a function only of the differences in eigenvalues $\theta_j - \theta_k$, with complete permutation symmetry as well. In the low-temperature, confining phase, we consider small fluctuations about P_0, defining $\theta_j = \theta_j^0 + \delta\theta_j$. Although this approximation may not be *a priori* valid, the assumption that fluctuations are small can be justified in the large-N limit. For small fluctuations

$$Tr_F P^k = \sum_{n=1}^{N} w^k z^{kn} e^{ik\delta\theta_n} \simeq \sum_{n=1}^{N} w^k z^{kn} ik\,\delta\theta_n \tag{12}$$

provided k is not divisible by N. Note that $Tr_F P^k$ takes the form of a discrete Fourier

transform in eigenvalue space. We define the Fourier transform of the fields ϕ_n as

$$\phi_k = \sum_{n=1}^{N} z^{kn} \delta\theta_n \tag{13}$$

so $Tr_F P^k = ikw^k \phi_k$. We will show below that the fields ϕ_k are the normal modes of the $Z(N)$-symmetric phase in a quadratic approximation. The mode $\phi_N \equiv \phi_0$ is identically zero for $SU(N)$, and from the reality of θ, we have $\phi_{N-n} = (\phi_n)^*$. In the case where k is divisible by N, $Tr_F P^k$ has a leading constant behavior of Nw^k, and the term linear in ϕ vanishes. The adjoint Polyakov loop operator is approximately $Tr_A P = Tr_F P \, Tr_F P^+ - 1 \simeq \phi_1 \phi_1^* - 1$. The operator $\phi_1 \phi_1^*$ has a non-zero vacuum expectation value, and should couple to scalar glueball states.

Operators with the same N-ality generally give inequivalent expressions when written in terms of the the ϕ variables. For example, $Tr_F P^k \propto \phi_k$ and $(Tr_F P)^k \propto (\phi_1)^k$. Group characters $\chi_R(P)$ are represented as sums of terms with the same N-ality, but with different mode content. For example, in $SU(4)$, the **10** and **6** representations are given by

$$\chi_{S,A} = \frac{1}{2} \left[(Tr_F P)^2 \pm Tr_F P^2 \right] \simeq \frac{1}{2} \left[\pm 2\phi_2 + i\phi_1^2 \right]. \tag{14}$$

As we discuss below in detail, these different combinations of fields will in general produce several different excitations, and only the lightest states will dominate at large distances.

The form of the effective action is fixed at high temperature by perturbation theory. The form of the effective action at high temperatures can be written as [30, 31]

$$S_{eff} = \beta \int d^3x \left[T^2 Tr_F \left(\nabla\theta \right)^2 + V_{1L}(\theta) \right]. \tag{15}$$

The kinetic term is obtained from the underlying gauge action via $\frac{1}{2} Tr_F F_{\mu\nu}^2 \rightarrow \frac{1}{2} 2 Tr_F \left(\nabla A_0 \right)^2 \rightarrow T^2 Tr_F \left(\nabla\theta \right)^2$. The potential $V_{1L}(\theta)$ is obtained from one-loop perturbation theory. For our purposes, it is conveniently expressed as [16, 17, 18]

$$
\begin{aligned}
V_{1L}(\theta) &= -\sum_{n=1}^{\infty} \frac{2}{\pi^2} \frac{T^4}{n^4} \left[|Tr P^n|^2 - 1 \right] \\
&= -\sum_{n=1}^{\infty} \frac{2}{\pi^2} \frac{T^4}{n^4} \left[N - 1 + \sum_{j \neq k} \cos\left(n \left(\theta_j - \theta_k \right) \right) \right].
\end{aligned}
$$

This series can be summed to a closed form in terms of the 4th Bernoulli polynomial. The complete one-loop expression has been obtained recently [32, 33, 34]; the complete kinetic term has a θ-dependent factor in front of the derivatives.

There are N equivalent solutions of the form

$$\theta_j^{(p)} = \frac{2\pi p}{N} \tag{16}$$

related by $Z(N)$ symmetry breaking. All of these solutions break $Z(N)$ symmetry, with $Tr_F P = N \exp\left(2\pi i p/N\right)$. For these values of θ, we recover the standard black-body result for the free energy.

A sufficiently general form of the action at all temperatures has the form

$$S_{eff} = \beta \int d^3x \left[\kappa T^2 Tr_F \left(\nabla\theta\right)^2 + V(\theta)\right] \tag{17}$$

where κ is a temperature dependent correction to the kinetic term, and V is a function only of the adjoint eigenvalues $\theta_j - \theta_k$. More complicated derivative terms can be added as necessary.

We assume that there is a finite free energy density difference associated with different values of P as $T \to 0$. Because the eigenvalues are dimensionless, this requires terms in the potential with coefficients proportional to $(mass)^4$ as $T \to 0$. We can expand the potential to quadratic order around P_0

$$V(\theta) \simeq V(\theta^0) + \sum_{j,k} \frac{1}{2} \left[\frac{\partial^2 V}{\partial\theta_j \partial\theta_k}\right]_{\theta^0} \delta\theta_j \delta\theta_k \tag{18}$$

where the coefficient in the expansion depends only on $|j - k|$. The quadratic piece is thus diagonalized by Fourier transform, and we can write

$$V(\theta) \simeq V(\theta^0) + \sum_{n=1}^{N-1} M_n^4 \phi_n \phi_{N-n}. \tag{19}$$

Similarly, the kinetic term becomes

$$\kappa T^2 Tr_F \left(\nabla\theta\right)^2 = \frac{\kappa T^2}{N} \sum_{n=1}^{N-1} \left(\nabla\phi_n\right)\left(\nabla\phi_{N-n}\right). \tag{20}$$

Once an ordering of eigenvalues is chosen, $Z(N)$ symmetry is expressed as a discrete translation symmetry in eigenvalue space. If we write the higher-order parts of S_{eff} in terms of the Fourier modes ϕ_n, each interaction will respect global conservation of N-ality. For example, in $SU(4)$, an interaction of the form $\phi_1^2 \phi_2$ is allowed, but not $\phi_1^2 \phi_2^2$.

The confining behavior of Polyakov loop two-point functions at low temperatures, for all representations of non-zero N-ality, is natural in the effective model. If the interactions

are neglected, we can calculate the behavior of Polyakov loop two-point functions at low temperatures from the quadratic part of S_{eff}. We have for large distances

$$\left\langle Tr_F P^n\left(x\right)Tr_F P^{+n}\left(y\right)\right\rangle \propto \left\langle \phi_n\left(x\right)\phi_n^*\left(y\right)\right\rangle \propto \exp\left[-\frac{\sigma_n}{T}\left|x-y\right|\right] \tag{21}$$

where $\sigma_n\left(T\right) = \sqrt{NM_n^4(T)/\kappa(T)}$ is identified as the string tension for the n'th mode at temperature T. Interactions may cause the physical string tensions to be significantly different from the tree-level result, but the general field-theoretic framework remains in any case. Of course, $\phi_{N-n} = \left(\phi_n\right)^*$ implies $\sigma_n\left(T\right) = \sigma_{N-n}(T)$. The number of different string tensions is $[N/2]$, the greatest integer less than or equal to $N/2$. The zero-temperature string tension is given at tree level by

$$\sigma_n^2\left(0\right) = NM_n^4(0)/\kappa(0) \tag{22}$$

This formalism gives a natural mechanism for the transition from Casimir scaling to scaling based on N-ality at large distances. As we have noted above, there are many composite operators with the same N-ality. For example, in $SU(8)$, the operators ϕ_1^4, $\phi_1^2\phi_2$, ϕ_2^2, $\phi_1\phi_3$, and ϕ_4 all have N-ality 4. Naively, the string tensions associated are $4\sigma_1$, $2\sigma_1 + \sigma_2$, $2\sigma_2$, $\sigma_1 + \sigma_3$, and σ_4, respectively. However, there is no symmetry principle prohibiting mixing of operators of the same N-ality. If there is mixing, then only the lightest string tension will be observed at large distances. Interactions between modes will lead to such mixing, and the operators $Tr_F P^k$ will exhibit a more complicated behavior.

Mixing between Polyakov loop operators of the same N-ality is easily understood using character expansion techniques applied to lattice gauge theories. For $d > 2$, there are strong-coupling diagrams in which the sheet between two Polyakov loops in a representation R split into a bubble with sheets of representation R_1 and R_2. Such strong-coupling graphs are non-zero when $R \subset R_1 \otimes R_2$; a necessary but not sufficient condition is that R and $R_1 \otimes R_2$ have the same N-ality. The same graph couples two representations R and R' of the same N-ality via the process $R \rightarrow R_1 + R_2 \rightarrow R'$. The case $d = 2$ is special: in the continuum, there are no gauge vector boson degrees of freedom, and the theory can be solved exactly, yielding Casimir scaling. The lattice theory is also exactly solvable; it reduces to a $d = 1$ spin chain model, and there are no bubble diagrams as in higher dimensions. Although the string tensions associated with a given lattice action do not in general give Casimir scaling, the fixed-point lattice action in two dimensions does yield results identical to the continuum[35].

It is easy to see that the $[N/2]$ string tensions $\sigma_1, \sigma_2, .., \sigma_{[N/2]}$ are all set independently within the class of effective models. A **minimal** model for the confined phase exhibiting this behavior is

$$V = \sum_{k=1}^{[N/2]} \frac{M_k^4}{k^2} Tr_F P^k Tr_F P^{+k} \tag{23}$$

where the M_k are arbitrary. The k'th term in the sum forces $Tr_F P^k = 0$, and gives rise to a mass for the mode ϕ_k.

Sine-law scaling arises naturally from a nearest-neighbor interaction in the space of Polyakov loop eigenvalues. Consider the class of potentials with pairwise interactions between the eigenvalues

$$V_2 = \sum_{j,k} v\left(\theta_j - \theta_k\right). \tag{24}$$

as obtained, for example, by two-loop perturbation theory[36]. An elementary calculation shows that at tree level

$$\sigma_n = \sqrt{\frac{2}{\kappa} \sum_{j=0}^{N-1} v^{(2)} \left(\frac{2\pi j}{N}\right) \sin^2 \left(\frac{\pi n j}{N}\right)} \tag{25}$$

where $v^{(2)}$ is the second derivative of v. This master formula relates the string tensions to the underlying potential. It is essentially the dispersion relation for a linear chain with arbitrary translation-invariant quadratic couplings: nearest-neighbor, next-nearest neighbor, *et cetera*. If the sum is dominated by the $j = 1$ and $j = N - 1$ terms, representing a nearest-neighbor interaction in the space of eigenvalues, then we recover sine-law scaling

$$\sigma_n \simeq \sqrt{\frac{4}{\kappa} v^{(2)} \left(\frac{2\pi}{N}\right)} \sin \left(\frac{\pi n}{N}\right). \tag{26}$$

There is a large class of potential which will give this behavior. $Z(N)$ scaling can be obtained by a very small admixture of other components of $v^{(2)}$, as we show below.

The string tension associated with different N-alities has been measured in $d = 3$ and 4 dimensions for $N = 4$ and 6, and in $d = 4$ for $N = 8$ [37, 38, 39, 40, 41]. We examine the simulation data by inverting equation (25) above to give a measure of the relative strength of the couplings $v^{(2)} \left(2\pi j/N\right)$. We normalize the result of this inversion such that the sum of the independent couplings adds to one, and the results are shown in Table I. Sine-law scaling corresponds to a value of 1 for $j = 1$, and 0 for the other $[N/2] - 1$ independent couplings. For $Z(N)$ scaling, the large-N limit gives $v^{(2)} \left(2\pi j/N\right) \propto 1/j^4$, yielding the result shown in

TABLE I: Relative strength of couplings $v^{(2)}(2\pi j/N)$

	d	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$SU(4)[37, 38]$	3	0.957(6)	0.043(2)		
$SU(4)[41]$	4	0.968(16)	0.032(7)		
$SU(4)[39, 40]$	4	0.992(25)	0.008(5)		
$SU(4)$ $Z(N)$	any	0.9412	0.0588		
$SU(6)[37, 38]$	3	0.930(5)	0.065(8)	0.004(5)	
$SU(6)[41]$	4	0.960(16)	0.045(18)	-0.005(12)	
$SU(6)[39, 40]$	4	0.996(40)	0.0003(218)	0.004(26)	
$SU(6)$ $Z(N)$	any	0.9266	0.0618	0.0116	
$SU(8)[41]$	4	1.028(22)	-0.067(24)	0.047(32)	-0.009(22)
$SU(8)$ $Z(N)$	any	0.9249	0.0583	0.0130	0.0037
$Z(N)$ $N \to \infty$	any	0.9239	0.0577	0.0114	0.0036
sine Law	any	1	0	0	0

the table. Note that the difference between sine-law scaling and $Z(N)$ scaling remains small but finite, even as N goes to infinity. The three-dimensional simulation results clearly favor $Z(N)$ scaling. The two sets of four-dimensional simulation results for $SU(4)$ and $SU(6)$ agree within errors, but one set lies systematically closer to the sine-law predictions, while the other set of results does not really favor either theoretical prediction. The $SU(8)$ results nominally favor the sine-law prediction. However, smaller error bars will be necessary to differentiate between sine-law and $Z(N)$ scaling in four dimensions.

The ultimate origin of confinement influences the form of the potential V, and the behavior of the different string tensions are derived in turn from V. In previous work on phenomenological models of the gluon equation of state[3, 5], we considered models of the form

$$f = -p = V(\theta) - 2 \int \frac{d^3k}{(2\pi)^3} Tr_A \ln \left[1 - P e^{-\beta \omega_k} \right] \tag{27}$$

where V is a phenomenologically chosen potential whose role is to favor confinement at low temperature. We studied two physically motivated potentials which reproduce $SU(3)$ thermodynamics well. Both give a second-order deconfining transition for $SU(2)$, and a first-order transition for $SU(N)$ with $N \geq 3$ in accord with simulation results. One potential is

a quadratic function of the eigenvalues

$$V_A = v_A \sum_{\alpha=2}^{N} \sum_{\beta=1}^{\alpha-1} (\theta_\alpha - \theta_\beta)(\theta_\alpha - \theta_\beta - 2\pi) \qquad (28)$$

which appears as an $O(m^2 T^2)$ term in the high temperature expansions at one-loop. This potential leads to $\sigma_k^A = \sigma_1$ for every N-ality. The other potential is the logarithm of Haar measure

$$V_B = v_B \sum_{\alpha=2}^{N} \sum_{\beta=1}^{\alpha-1} \ln\left[1 - \cos(\theta_\alpha - \theta_\beta)\right]. \qquad (29)$$

which is motivated by the appearance of Haar measure in the functional integral. This term is cancelled out in perturbation theory for flat space[17], but not in other geometries[29]. The potential V_B was first studied by Dyson in his fundamental work on random matrices[42], and leads to

$$\sigma_k^B = \sqrt{\frac{k(N-k)}{N-1}} \sigma_1 \qquad (30)$$

which one might call "square root of Z(N) scaling". Although these two models are not consistent with the lattice simulation data for σ_k for $N > 3$, they remain viable phenomenological forms for $N = 2$ and 3. It is interesting to note that there is a well-studied potential which gives $Z(N)$ scaling[43]. It is the integrable Calogero-Sutherland-Moser potential

$$V_Z(\theta) = \sum_{j \neq k} \frac{\lambda}{\sin^2\left(\frac{\theta_j - \theta_k}{2}\right)} \qquad (31)$$

which has been associated with two-dimensional gauge theory[19, 20].

The effective action has domain wall solutions in the deconfined phase, generalizing the behavior seen in the perturbative, high-temperature form of the effective action[30, 31]. In this limit, the $Z(N)$ symmetry breaks spontaneously, with N equivalent vacua characterized by $P = z^n I$. There is a surface tension associated with one-dimensional kink solutions of the effective equations of motion which interpolate between different phases. There are $\left[\frac{N}{2}\right]$ different surface tensions ρ_k, each associated with the kink solution connecting the $n = 0$ phase with the $n = k$ phase. Giovannangeli and Korthals Altes[44] have given an argument valid at high temperature indicating that the surface tensions ρ_k obey $Z(N)$ scaling. This argument can be extended with minor modifications to arbitrary potentials of the form V_2, giving semiclassically

$$\rho_k = \frac{k(N-k)}{N-1} \rho_1. \qquad (32)$$

in the entire deconfined phase.

A similar argument has been given for the string tension in the confined phase of $2 + 1$-dimensional Polyakov models, which consist of gauge fields coupled to scalars in the adjoint representation[45]. In this case, the effective potential has the form V_2 when written in terms of dual variables. It is natural to speculate that there is a class of self-dual effective models in two spatial dimensions with identical string-tension scaling laws in both phases.

We have constructed the most general class of effective actions which can be built from the eigenvalues of the Polyakov loop. $Z(N)$ symmetry requires a symmetric distribution of eigenvalues in the confined phase, leading to a special role for $Z(N)$ Fourier modes. Mode mixing provides a natural crossover from Casimir scaling to scaling based on N-ality. There is a natural association between sine-law scaling and strong nearest-neighbor interactions between eigenvalues, and sine-law scaling and $Z(N)$ scaling are in some sense quite close. Nevertheless, string tensions in different N-ality sectors are determined by the parameters of the effective action, and are *a priori* arbitrary. It is clear from studies of different lattice gauge actions in two dimensions that the ratios of lattice string tensions need not be universal. Only near the continuum limit are two-dimensional string tension ratios universal. It remains mysterious why the wide class of models so far studied have not provided us with a wider variety of string tension scaling laws. We do not know if additional fields of zero N-ality can change string tension scaling in the continuum, and there has not been much exploration of the possible effect of fields that preserve a non-trivial subgroup of $Z(N)$. Studies of such models might give insight into the connection between confinement mechanisms and string-tension scaling laws.

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